Seat No. _______________

HC-003-1164004

M. Sc. (Sem. IV) Examination

April - 2023

CMT-4004 : Mathematics

(Graph Theory)

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) There are total five questions.
(3) Each question carries equal marks (14).

1 Answer any **seven** questions : **7×2=14**

(1) Define terms:
  (1) Self loop
  (2) Parallel edges and
  (3) Simple graph

(2) Draw a simple graph on three vertices and a graph with self loop as well as parallel edges on two vertices.

(3) Define terms:
  (1) Odd vertex and
  (2) Even vertex

(4) Define term:
  (1) Pendent vertex and
  (2) Pendent edge

(5) Define term: Tree and draw a tree on five vertices with indication of its all pendent vertices as well as pendent edges.

(6) Define term: Minimally connected graph. Also draw a minimally connected graph on two vertices.

(7) Define terms:
  (1) Hamiltonian cycle
  (2) Hamiltonian path and
  (3) Hamiltonian graph

(8) Define term: Cut-set and draw a simple graph on four vertices with indication of its two or three cut-sets.

(9) Define terms:
   (1) Fundamental cycle and
   (2) Fundamental cut-set

(10) Define terms:
   (1) Weighted graph and
   (2) Minimal spanning tree.

2 Answer any two questions: **2×7=14**

(a) Let $G=(V,E)$ be a graph. Prove that, G is a disconnected graph if and only if there are two disjoint subsets V_1 and V_2 of V such that, (i) $V=V_1\cup V_2$ and (ii) there is no edge uv in G, whose one end vertex lies in V_1 and another end vertex lies in V_2.

(b) Let G be a simple graph with n vertices, q edges and k number of components in G. Prove that, $q\le\frac{1}{2}(n-k)(n-k+1)$.

(c) Let G be be connected grpah. Prove that, G is an Euler grpah $\Rightarrow d_G(v)=\text{even},\ \forall, v\in V(G)$.

3 Answer following two questions: **2×7=14**

(1) Let G be a simple graph and G has atleast two vertices. Suppose $d_G(v)\ge\frac{n}{2}, \forall v V(G)$. Prove that G, is a Hamiltonian graph.

(2) Let G be a connected graph. Prove that, G is an open Euler graph if and only if G has precisely two odd vertices and remaining are even vertices.

OR

3 Answer following two questions: **2×7=14**

(1) Let G be an acyclic graph with n vertices and k components. Prove that, G has $n–k$ edges.

(2) Let G be a simple graph on n vertices, Let $u, v \in V(G)$ be two non-adjacent vertices of G such that, $d_G(u) + d_G(v) \geq n$. Prove that, G is Hamiltonian graph if and only if its super graph $G + \{uv\}$ is Hamiltonian.

4 Answer following two questions: **2×7=14**

(a) For a tree T, with $|V(T)| = n$, prove that, T has $n–1$ edges.

(b) Prove that, a graph G is a minimally connected graph if and only if it is a tree.

5 Answer any two questions: **2×7=14**

(i) Let T be a tree with $|V(T)| \geq 2$. Prove that, T is a 2-chromatic graph.

(ii) Define adjacency matrix for a graph G. Write down adjacency matrix for C_6. Also write down atleast four properties for the adjacency matrix $X(G)$, for a graph G.

(iii) For a simple connected planner graph G, derive Euler's formula $f = e - n + 2$.

(iv) Let T be a tree with n vertices $(n \geq 2)$. Prove that, T has either one center or two centers. Also prove that, in the case of T admits two centers, they must be adjacent by an edge in T.